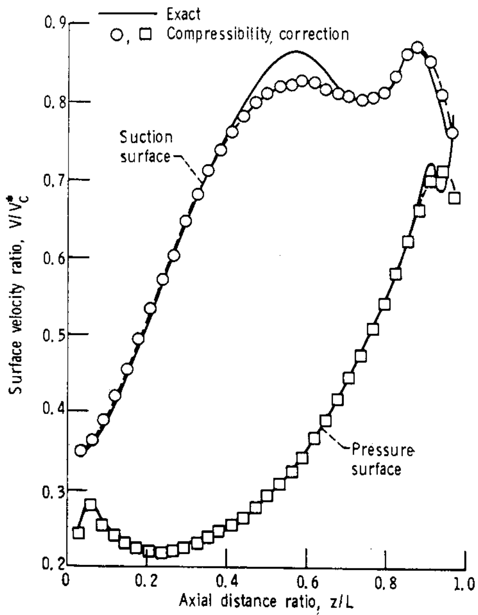


**Fig. 3 Comparison of exact and compressibility correction values of surface velocity. Average compressible density from exact compressible density profile.**

except for the region of the first velocity peak on the suction surface.

For the general case for which the exact compressible values are unknown, the average compressible density can be determined from the average incompressible velocity as follows. From continuity

$$\bar{V}_i/V_c^* = (\bar{\rho}_c/\rho_t)(\bar{V}_c/V_c^*) \quad (2)$$

where $V_c^*$ is the critical velocity. For isentropic flow, the compressible critical velocity ratio can be expressed in terms of the density ratio

$$\bar{V}_c/V_c^* = \{[(\gamma + 1)/(\gamma - 1)][1 - (\bar{\rho}_c/\rho_t)^{\gamma-1}]\}^{1/2} \quad (3)$$

which, when combined with Eq. (2), yields

$$\bar{V}_i/V_c^* = (\bar{\rho}_c/\rho_t)\{[(\gamma + 1)/(\gamma - 1)][1 - (\bar{\rho}_c/\rho_t)^{\gamma-1}]\}^{1/2} \quad (4)$$

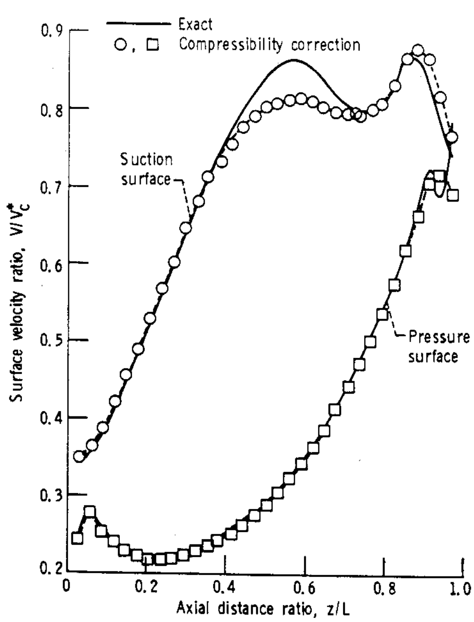


**Fig. 4 Comparison of exact and compressibility correction values of surface velocity. Average compressible density from average incompressible velocity [Eq. (4)].**

which now relates the average compressible density to the average incompressible velocity.

The comparison between the exact compressible velocities and the correction values with $\bar{\rho}_c$ obtained from Eq. (4) is shown in Fig. 4. Again, the agreement is considered excellent, except for the region of the first velocity peak on the suction surface.

The encouraging results obtained in the comparisons of Figs. 3 and 4 strongly suggest further investigation of the applicability of the devised compressibility correction for internal flow solutions.

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# Internal Parachute Flows

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### 1. Introduction

THERE has been little success in theoretically predicting details of flowfields surrounding porous parachute canopies. The basic design objective of taking full advantage of fluid viscosity and use of a permeable material requires most analyses to be in terms of average quantities and heavily dependent upon empirical results. As a first step away from this approach, it can be asked if a potential flow model might reasonably predict some portion of the flowfield. One such method is described here with application to the steady incompressible flow internal to a porous canopy of arbitrary axisymmetric cross section.

### 2. Model

The particular model chosen was that of a vortex sheet placed to coincide with the physical location of the canopy. In cylindrical coordinates the Stoke's stream function corresponding to an axisymmetric vortex sheet placed in a uniform stream of velocity $-U$ is[1]

$$\psi = (-r/4\pi)\int_0^{2\pi}\cos\theta d\theta \int_0^{S_{max}} \gamma r_i[r^2 + r_i^2 + (z - z_i)^2 - 2rr_i\cos\theta]^{-\frac{1}{2}}ds_i + Ur^2/2$$

The $i$ subscript refers to a sheet location,

$$ds_i = (dr_i^2 + dz_i^2)^{\frac{1}{2}}$$

and $\gamma = \gamma(s_i)$ is the unknown local sheet strength. Determination of this quantity is made algebraically. The canopy

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is replaced with a set of conical frustrums. This, together with taking $\gamma$ constant over each frustrum, allows analytical integration over $s_i$. Formation of the radial and axial velocities from the stream function, application of the no-normal flow boundary condition at some intermediate point of each frustrum and numerical integration over $\theta$ provides a set of simultaneous algebraic equations in the sheet strength. (Note that assuming the $\gamma$ to be sectionally constant does not provide the free constant usually found in vortex sheet applications. Consequently, there is no opportunity to impose a Kutta condition or some facsimile.) Discrete application of the boundary condition provides for some flow through the sheet, thereby duplicating a porous material.

Once the $\gamma$ distribution is obtained, velocities anywhere in the field except on the sheet itself may be found from the appropriate $\psi$ derivatives. The surface values would come from the $\psi$ derivatives $\pm\gamma/2$.

### 3. Application

An indication of the porosity of the mathematical "fabric" may be obtained from application to a solid hemispherical parachute. The calculated mass flow per unit time and area between two points near the portion of the canopy oriented at a 45° angle to the freestream is very nearly the same function of differential pressure for $\Delta p < 100$ psf as is a specimen of MIL-C-7020 TYPE III (Ref. 2). The effective porosity for the entire canopy, defined as the ratio of the average velocity through the cloth to the freestream velocity is 0.042. Results are not significantly different using 24 frustrums than they are for only 12.

Measured[3] and calculated internal surface pressures are shown in Fig. 1 for O'Hara models[4] 3, 5 and 7. There was no agreement between external surface pressure distributions due to flow separation. Mouth plane axial velocities are given for the same geometries in Fig. 2, with the experimental values being obtained through pressure rake surveys.[4] The large variations in computed values toward the skirts are due to the proximity to the edge of the vortex sheet. Vent plane axial velocities[4] agreed well qualitatively, increasing from the centerline values to ones roughly one-third greater at the vent boundary, but the calculated magnitudes were greater by factors of 3 to 4. The measured velocities were somewhat suspect, however, as in no case did they satisfy the conservation of mass. Substitution of the calculated vent plane values would bring measured vent and fabric mass outflows close to the measured inflows.

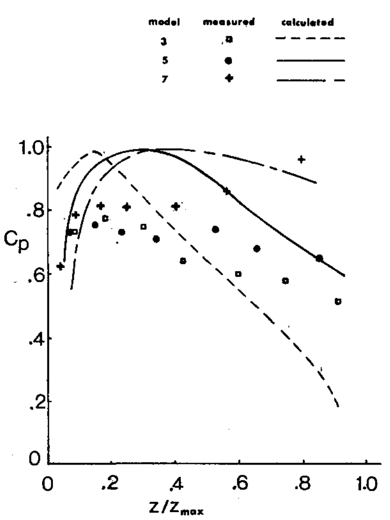


**Fig. 1 Internal surface pressure distributions on O'Hara models 3, 5, and 7.**

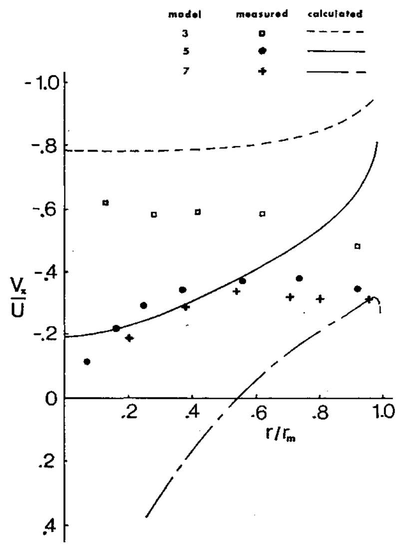


**Fig. 2 Mouth plane axial velocities for O'Hara models 3, 5, and 7.**

### 4. Concluding Remarks

These theoretical predictions of internal flow properties are only fair numerically, but they represent general behavior reasonably well. There may be applications where results of this type are more desirable or more available than average or empirically derived quantities.

Although results may be improved by assuming a variable sheet strength distribution, no essential gains in applicability could be expected due to the failure to consider viscosity. Significant advances might be made by adapting the technique to the opening problem. Even in highly viscous fluids, impulsive motions produce flowfields which closely resemble potential flows. The relatively short times involved in the opening problem suggest that an inviscid approximation may be a very realistic one.

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# Study of Rotating Airfoil

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### Introduction

THE performance of a rotating airfoil depends upon the effects produced due to rotation. Experiments conducted by H. Himmelskamp[1] show the large rotational effects exist near the hub. Himmelskamp assumed these effects to be

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